DOCUMENT RESUME

ED 074 156 TM 002 521

AUTHOR	Hofmann, Richard J.
TITLE	The Computation of Orthogonal Independent Cluster Solutions and Their Oblique Analogs in Factor Analysis.
PUB DATE	Mar 73
NOTE	24p.; Paper presented at annual meeting of the American Educational Research Association (New Orleans, Louisiana, February 25-March 1, 1973)
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*Cluster Analysis; *Comparative Analysis; *Factor Analysis; Factor Structure; Models; Research Methodology; Speeches; Statistical Analysis; Technical Reports

ABSTRACT

A very general model for the computation of independent cluster solutions in factor analysis is presented. The model is discussed as being either orthogonal or oblique. Furthermore, it is demonstrated that for every orthogonal independent cluster solution there is an oblique analog. Using three illustrative examples, certain generalities are made with respect to the class of independent cluster solutions which are generated by the orthomax criterion. A procedure is presented for comparing the simple structure of an orthogonal independent cluster solution with the simple structure of its oblique analog. (Author)

25.17

The Computation of Orthogonal Independent Cluster Solutions and their Oblique Analogs in Factor Analysis

Richard J. Hofmann

Miami University

Paper Prensented at Annual Meeting of

American Educational Research Association,

New Orleans, March 1973.

In factor analysis one generally thinks of a transformation solution within either an orthogonal or an oblique framework. In this manuscript a general model is presented for the independent cluster solution in factor analysis. It is demonstrated that there may possibly be a variety of independent cluster solutions which may be generated for a given set of data. Every orthogonal independent cluster solution is discussed as having an oblique analog.

The equations presented are based upon the Harris and Kaiser[1964] theory of developing oblique solutions through the use of orthogonal transformation matrices. An analytic basis for the computation of the orthogonal transformation matrices is presented in the form of the orthomax criterion[Saunders, 1962] and finally some empirical applications of the model are presented and discussed.

Assume some ($\underline{n} \times \underline{r}$) factor matrix $\underline{P}$ determined by any orthogonal factoring procedure. Then let the major product moment of $\underline{P}$ define an ($\underline{n} \times \underline{n}$) singular matrix, $\underline{R}^*$, of rank $\underline{r}$.

(1) $\underline{R}^* = \underline{PP}'$

Assuming that the initial factoring was done on a population variance-covariance matrix or correlation matrix, the diagonal entries of $\underline{R}^*$ will represent the common variances of the variables while the $\underline{ij}$ th off-diagonal entry of $\underline{R}^*$ will represent the covariance between the common portions of variables $\underline{i}$ and $\underline{j}$ and will approximate the population covariance between variables $\underline{i}$ and $\underline{j}$.

The principal axis representation of $\underline{R}^*$ will provide a basis for the discussion to follow. Compute the principal axis representation $\underline{QM}$ of $\underline{R}^*$, where the columns of $\underline{Q}$ are unit length latent vectors associated with the non-zero latent roots which are the diagonal entries of $\underline{M}^2$.

The General Model for an Independent Cluster Solution of R^*

Let the ($\underline{r}$ x $\underline{r}$) square matrix $\underline{T}$ represent any orthonormal matrix; then ($\underline{T}'\underline{T} = \underline{TT}' = \underline{I}$). All orthogonal transformation solutions, $\underline{F}$, of $\underline{QM}$ may be depicted as

(2) $\underline{F} = \underline{QMT}$. (An orthogonal transformation solution of $\underline{R}^*$.)

Thus the basic structure of $\underline{F}$ consists of a left orthonormal, $\underline{Q}$, a right orthonormal, $\underline{T}$, and a basic diagonal $\underline{M}$. The matrix $\underline{R}^*$ represents the major product moment of $\underline{F}$. In a later section the computation of $\underline{T}$ is discussed in some detail. At this point it is only necessary to assume that once determined $\underline{T}$ is constant in equations 2, 3, 4, 5, 6, 7, and 8.

For the oblique analog of $\underline{F}$, let the ($\underline{n}$ x $\underline{r}$) matrix $\underline{A}^*$ represent a primary pattern matrix, in the sense of Holzinger and Harman [1941] and let $\underline{B}^*$ represent a primary structure matrix with $\underline{W}^*$ representing the the intercorrelations of the primaries. Then from Harris and Kaiser [1964] the following equations may be used for computing an independent cluster solution :

(3) $\underline{A}^* = \underline{QTD}$; (An Oblique Primary Pattern for $\underline{R}^*$.)

(4) $\underline{W}^* = \underline{D}^{-1}\underline{T}'\underline{M}^2\underline{TD}^{-1}$; (An Oblique Primary Intercorrelation Matrix.)

(5) $\underline{B}^* = \underline{QM}^2\underline{TD}^{-1}$. (An Oblique Primary Structure for $\underline{R}^*$.)

The matrix $\underline{D}$ in equations 3, 4, and 5 is a diagonal matrix totally dependent upon the values of $\underline{M}$ and $\underline{T}$ and is chosen so that ($\underline{MT}$) will be column normalized when post multiplied by $\underline{D}^{-1}$, thereby insuring that the primaries are of unit variance. If $\underline{D}$ and its inverse are eliminated from equations 3, 4, and 5 then the oblique covariance matrices result:

(6) $\underline{A} = \underline{QT}$;

(7) $\underline{W} = \underline{T}'\underline{M}^2\underline{T}$;

(8) $\underline{B} = \underline{QM}^2\underline{T}$.

The matrix $\underline{A}$ may be thought of as a matrix of raw regression weights for predicting the $\underline{n}$ variables from the $\underline{r}$ un-normalized primaries. The matrix $\underline{W}$ represents the covariances between the $\underline{r}$ primaries with the diagonal of $\underline{W}$ representing the variances of the $\underline{r}$ primaries and also representing the diagonal elements of $\underline{D}^2$. The matrix $\underline{B}$ may be thought of as a matrix of covariances of the $\underline{r}$ primaries with the $\underline{n}$ variables.

Within a Thurstonian [1947] framework the matrix $\underline{A}$ may also be thought of as the matrix of covariances of the $\underline{r}$ reference vectors with the $\underline{n}$ variables. While the matrix $\underline{B}$ may be thought of within a Thurstonian framework as the matrix of raw regression weights for predicting the $\underline{n}$ variables from the $\underline{r}$ un-normalized reference vectors. The inverse of $\underline{W}$, $\underline{W}^{-1}$, represents the covariances between the $\underline{r}$ reference vectors with the diagonal of $\underline{W}^{-1}$ representing the variances of the $\underline{r}$ reference vectors.

The well known relationships between pattern, structure, factor intercorrelation and $\underline{R}^*$ may be denoted as:

(9) $\underline{R}^* = \underline{AWA}' = \underline{BA}' = \underline{A^*W^*A^{*\prime}} = \underline{B^*A^{*\prime}}$.

Regardless of whether one is working in a Thurstone or a Holzinger framework the matrix of interest in the oblique solution depicted here is $\underline{A}$. In equation 7 the matrix $\underline{W}$ represents the minor product moment of $\underline{F}$. Thus $\underline{R}^*$ and $\underline{W}$ are product moment matrices determined from the same "basic structure". The orthonormal of $\underline{R}^*$ is a left orthonormal for $\underline{F}$ and the orthonormal of $\underline{W}$ is a right orthonormal for $\underline{F}$. The latent roots of $\underline{W}$ are identical to the latent roots of $\underline{R}^*$. The matrix $\underline{A}$ is the product of the left and right orthonormals of $\underline{F}$.

If theoretically "true" independent clusters are defined by the data then each row of the "factor matrix" will contain only one non-zero entry. Each variable will be of complexity one and the planar plot of any pair of columns of $\underline{QM}$ will show a concentration of points along two radial streaks; a number of points at the origin and no points off the radial streaks. If the independent clusters are mutually orthogonal $\underline{R}^*$ will be a diagonal super matrix if the variables are ordered properly and will have zero entries for all matric elements off the main diagonal. The principal axis solution for such a matrix, $\underline{QM}$, will have ideal simple structure (Harris and Kaiser, 1964). If the independent clusters defined by $\underline{R}^*$ are not mutually orthogonal then $\underline{R}^*$ will not have the form of a diagonal supermatrix. For oblique independent clusters the radial streaks defined by $\underline{QM}$ may be "orthogonalized" by post-multiplying $\underline{QM}$ by $\underline{M}^{-1}$. For perfect oblique independent clusters the planar plot of any pair of columns of $\underline{Q}$ will show a concentration of points along two <u>orthogonal</u> radial streaks; a number of points at the origin and no points off the radial streaks. The $\underline{r}$ radial streaks defined by $\underline{Q}$ will be mutually orthogonal. For non-independent cluster data $\underline{Q}$ will define a scattering of points but not radial streaks.

Variance Allocation in the Independent Cluster Solution

In describing the allocation of common variance there is an interesting duality between the oblique and the orthogonal independent cluster solutions. Following from equation 9, and Holzinger and Harman (1941, p.247) let the common variance of the $\underline{j}$ th variable, $\underline{h}_j^2$, be expressed in the general terms of an oblique solution as follows:

$$(10)\quad \underline{h}_j^2 = \underline{a}_{j1}^{*2} + \underline{a}_{j2}^{*2} + \ldots + \underline{a}_{jr}^{*2} + 2\underline{a}_{j1}^{*}\underline{a}_{j2}^{*}\underline{w}_{12}^{*} + \ldots + 2\underline{a}_{j,r-1}^{*}\underline{a}_{j,r}^{*}\underline{w}_{r-1,r}^{*}.$$

The direct contributions of the factors to the common variance of variable $\underline{j}$ are given by the first $\underline{r}$ terms of equation 10, while the joint contributions of the factors are given by the remaining, $\underline{r}(\underline{r}-1)/2$, terms in equation 10.

The direct contributions of the $\underline{r}$ factors to the common variance of all $\underline{n}$ variables are obtained by summing the direct contributions of each factor across all $\underline{n}$ variables as follows:

$$(11)\quad \sum_{j=1}^{n} a_{j1}^{*2}\ ,\quad \sum_{j=1}^{n} a_{j2}^{*2}\ ,\ \ldots\ ,\ \sum_{j=1}^{n} a_{jr}^{*2}\ .$$

The $\underline{r}(\underline{r}-1)/2$ joint contributions of pairs of factors across all $\underline{n}$ variables are given by:

$$(12)\quad 2\underline{w}_{12}^{*}\sum_{j=1}^{n}\underline{a}_{j1}^{*}\underline{a}_{j2}^{*}\ ,\quad 2\underline{w}_{13}^{*}\sum_{j=1}^{n}\underline{a}_{j1}^{*}\underline{a}_{j3}^{*}\ ,\ \ldots,\ 2\underline{w}_{r,r-1}^{*}\sum_{j=1}^{n}\underline{a}_{jr}^{*}\underline{a}_{j,r-1}^{*}\ .$$

In expression 11 the values determined are just the column sums of squares for $\underline{A}^*$ which are given also as the diagonal elements of the minor product of $\underline{A}^*$. However the diagonal elements of the minor product of $\underline{A}^*$ are just the $\underline{r}$ diagonal elements of $\underline{W}$ or the variances of the $\underline{r}$ primaries.

$$(13\text{-a})\quad (\underline{A}^{*})'(\underline{A}^{*}) = \underline{DT'Q'QTD} = \underline{D}^{2} \quad ;$$

$$(13\text{-b})\quad \underline{D}^{2} = \text{diagonal } [\, \underline{T'M^{2}T} \,] \, .$$

Therefore within an oblique independent cluster framework the direct contributions of the factors as noted in expression 11 may be written as follows:

$$(14)\quad \sum_{j=1}^{n} \underline{a}_{j1}^{*2} = \underline{w}_{11},\ \sum_{j=1}^{n} \underline{a}_{j2}^{*} = \underline{w}_{22}, \ldots, \sum_{j=1}^{n} \underline{a}_{jr}^{*2} = \underline{w}_{rr} \, .$$

The total joint contribution for the pair of factors $\underline{k}$ and $(\underline{k}-1)$ as given by expression 12 is

$$2\underline{w}^{*}_{k,k-1} \sum_{j=1}^{n} \underline{a}^{*}_{jk}\ \underline{a}^{*}_{j,k-1}$$

but the element $(\sum_{j=1}^{n} \underline{a}^{*}_{jk}\ \underline{a}^{*}_{j,k-1})$ is just the ($\underline{k}$, $\underline{k}$-1)th off-diagonal entry of $\underline{D}^2$, which is zero, thereby implying that the following equality

$$(15)\quad 2\underline{w}^{*}_{k,k-1} \sum_{j}^{n} \underline{a}^{*}_{jk}\ \underline{a}^{*}_{j,k-1} = 0$$

will always hold for the oblique independent cluster solution. That is to say, regardless of the "obliquity" of an oblique independent cluster solution the total joint contribution of any pair of factors to the total variance will always be zero.

It is immediately apparent from expression 11 that for an orthogonal independent cluster solution there are no joint contributions for the factors inasmuch as all nondiagonal values of $\underline{W}^{*}$ are zero. The direct contributions of the factors are just the column sums of square for $\underline{F}$. Thus the total sum of squares within $\underline{A}^{*}$ is the same as the total sum of squares within $\underline{F}$.

It would seem that the orthogonal and oblique independent cluster solutions represent the extreme ends of a span of possible oblique solutions for a given set of data. For the oblique independent cluster all common variance may be accounted for totally within the framework of second order factors, the diagonal of $\underline{W}$. For the orthogonal independent cluster solution, the common variance contributions are accounted for solely as a function of the uncorrelated first order factors. Thus, those oblique solutions falling between these two independent cluster solutions would utilize both first and second order factors to account for common variance.

Computation of the Orthonormal T for Use in the Independent Cluster Model

Although the model discussed in this paper may be generalized to any square ($\underline{r}$ x $\underline{r}$) orthonormal, only the class of analytic orthonormals associated with the orthomax criteria will be discussed. Saunders(1962) was the first to note that all analytic orthogonal transformation procedures could be combined for general discussion inasmuch as they all involve the same fourth degree moments of the data.

The general orthomax transformation criterion involves a maximization of

$$(16) \quad \text{Maximum} = \sum_{k=1}^{r} [\, \underline{n} \sum_{j=1}^{n} \underline{f}_{jk}^{4} - \underline{s}(\sum_{j=1}^{n} \underline{f}_{jk}^{2})^{2}]$$

Where $\underline{f}_{jk}$ is the $\underline{jk}$ th element of the transformation solution, when the criterion is applied to a factor matrix whose rows are normalized , the solution is referred to as a "normal" - type solution. When the transformation solution is normal $\underline{f}_{jk}$ is the $\underline{jk}$ th element of the matrix ($\underline{H}^{-1}\underline{QMT}$). The matrix $\underline{H}^2$ is a diagonal matrix whose nonzero elements represent the variance of the $\underline{n}$ variables and is the diagonal of $\underline{R}^*$. When the criterion is applied to a factor matrix whose rows are not normalized the solution is referred to as a "raw"-type solution.

When the transformation solution is raw f_{jk} is the jk th element of the matrix (QMT).

The value of s in equation 16 is most crucial as it is the "orthomax weight" and it specifically defines the blind orthogonal transformation that is computed. When the orthomax weight is zero, ($s = 0$), maximizing (QMT) in equation 16 will result in the raw quartimax transformation [Neuhaus and Wrigley, 1954]. When the orthomax weight is unity, ($s = 1$), equation 16 defines Kaiser's[1958] varimax criterion. Within this same framework it is possible to define Saunder's[1962] equamax criterion, ($s = r/2$), and, theoretically, the principal axis transformation criterion, ($s = -\infty$) [Kaiser,1966].

Harris and Kaiser[1964] in their discussion of the independent cluster solution suggest that the orthomax criterion be applid to (QT) rather than (QMT). Furthermore they note that because it is Q, the unit length latent vectors of R^*, that are being transformed the second term in equation 16 will be a constant and is therefore irrelevant in the transformation process. Thus, Harris and Kaiser always compute T to implicitly maximize the raw quartimax criterion applied to (QT). Although Harris and Kaiser do not note it specifically it is possible to utilize the transformation matrix T computed as a function of Q to define an orthogonal solution of the form (QMT). Technically then for this one procedure the oblique solution has an orthogonal analog. In the empirical section to follow we will modify the Harris and Kaiser procedure just slightly by using the orthonormal T computed as a function of (Q) to define an orthogonal solution of the form (QMT). The set of solutions computed in this fashion will be referred to as the "Harris and Kaiser indpendent cluster solutions". All discussion in the previous sections is perfectly generalizable to the Harris and Kaiser independent cluster solutions.

Empirical Applications of the General Independent Cluster Model

To clarify and illustrate the independent cluster model presented in this manuscript six examples of independent cluster solutions are discussed. Five of these examples are discussed within the framework of illustrative data. For each of the illustrative examples the solution development started with the principal axis representation of some orthogonal factor solution.

For each data set orthogonal and oblique independent cluster solutions were computed. One set of solutions was computed using the modified Harris and Kaiser procedures. Twenty-four additional solutions were generated through a systematic variation of the orthomax weight in equation 16. The orthomax weight was varied from negative to positive three in increments of one-half; first for the raw matrix ($\underline{QM}$) and then for the row normalized matrix ($\underline{H}^{-1}\underline{QM}$). With two exceptions only the "best" set of independent cluster solutions is presented along with the Harris and Kaiser solutions for each problem.

Example 1 Perfect Orthogonal Independent Clusters

As previously noted when perfect orthogonal independent clusters are defined by the data the principal axis representation of the factor solution will have ideal simple structure. In such a case given any one of the family of orthomax criterion depicted by equation 16 the only transformation of ($\underline{Q}$) or ($\underline{QM}$) that will maximize it is the identity matrix, $\underline{T} = \underline{I}$. For such a data set, then ($\underline{QMT}$) and ($\underline{QTD}$) will be identical. (Because of the redundancies of solutions for such an example we will not provide any numerical representations of it.)

Example 2 Perfect Oblique Independent Clusters(Bipolar-Plasmodal)

For at least one data set defining perfect oblique independent clusters there is a possible problem in maximizing the raw orthomax criterion when applied to ($\underline{QMT}$). In Table 1 the principal axes form of example 2 is presented. Note that when the orthomax criterion, ($.5 \geq \underline{s} \geq -3$), is applied to ($\underline{QMT}$) for this example the maximization of the criterion occurs for, ($\underline{T} = \underline{I}$), the identity matrix. As may be seen in Table 1 a raw orthomax solution when used in conjunction with the general independent cluster solution may result in a misleading orthogonal representation of data that define perfect oblique clusters.

Table 1 about here

Presented also in Table 1 are the Harris and Kaiser independent cluster solutions for the data set. Clearly the oblique independent clusters define an excellent transformation solution for these data.

Finally in Table 1 the normal varimax independent cluster solutions are provided. There solutions are identical to the Harris and Kaiser solutions. For these solutions the matrix ($\underline{H}^{-1}\underline{QMT}$) was utilized in maximizing the orthomax criterion with the orthomax weight equal to unity. It is both interesting and informative to point out that for all orthomax weights greater than or equal to unity the matrix ($\underline{H}^{-1}\underline{QMT}$)

ble 1 Selected General Independent Cluster Solutions for Illustrative Example 2*.

		Type Solution										
				Harris and Kaiser				Raw Orthomax[1]		Normal Varimax[2]			
		Principal Axis		Oblique		Orthogonal		Oblique/Orthogonal		Oblique		Orthogonal	
Pattern Matrix	1	70	20	73	00	64	-35	-70	20	73	00	64	-35
	2	-70	-20	-73	00	-64	35	70	-20	-73	00	-64	35
	3	-70	20	00	73	-35	64	70	20	00	73	-35	64
	4	70	-20	00	-73	35	-64	-70	-20	00	-73	35	-64
Primary Intercorrelations				100	-85			100	00	100	-85		
				-85	100			00	100	-85	100		
Associated Orthogonal Transformation Matrix				71	-71	71	-71	100	00	71	-71	71	-71
				71	71	71	71	00	100	71	71	71	71

* All values have been multiplied by 100 to eliminate decimals.

1 This solution was associated with all orthomax solutions computed for the weights negative three through positive one half inclusive.

2 This solution was associated with all orthomax solutions computed for the weights positive one through positive three inclusive.

would always maximize the orthomax criterion utilizing the same orthonormal $\underline{T}$. However for orthomax weights less than unity the only orthonormal $\underline{T}$ that would provide a maximization for the orthomax criterion applied to ($\underline{H}^{-1}\underline{QMT}$) was the identity matrix.

The solutions in Table 1 suggest that perhaps one would be well advised not to utilize raw transformation solutions with the general independent cluster model, at least initially.

Example 3 Approximate Oblique Independent Clusters (Non-Plasmodal)

The data used for this example define oblique factors that represent approximate independent clusters. These data are the eight physical variables referred to by Holzinger and Harman(1941) and used as an illustrative example by Harris and Kaiser(1964) in the development of their independent cluster solution.

Table 2 about here

For these particular data there were a number of independent cluster solutions generated by the orthomax criterion that did define clear structure. However all of those solution sets associated with orthomax weights greater than or equal to unity defined essentially the same solutions; being almost identical to the solution set defined by the Harris and Kaiser independent cluster procedures.

Although all 24 orthomax solutions generated for this data set are not presented here they appeared to define a very systematic relationship between the orthomax weight, the orthogonal transformation matrix and the primary intercorrelation matrix. As the orthomax weight was varied from

Table 2 Selected General Independent Cluster Solutions for Illustrative Example 3*.

		Type Solution									
				Harris and Kaiser				Normal Varimax			
		Principal Axis†		Oblique		Orthogonal		Oblique		Orthogonal	
Pattern Matrix	1	86	-33	89	05	88	26	89	07	88	27
	2	85	-41	96	-03	92	20	95	-02	92	21
	3	81	-41	93	-05	89	17	93	-04	89	18
	4	83	-34	88	03	86	24	87	04	86	25
	5	75	56	01	93	25	90	-01	94	24	90
	6	64	51	-02	82	19	79	-04	84	18	79
	7	56	49	-06	77	14	73	-07	77	14	73
	8	62	38	08	69	26	68	07	69	25	68
Primary Intercorrelations				100	48			100	48		
				48	100			48	100		
Associated Orthogonal Transformation Matrix				79	-61	79	-61	78	-62	78	-62
				61	79	61	79	62	78	62	78

* All values have been multiplied by 100 to eliminate decimals.

† Computed from centroid solution.

negative three to positive three the primary intercorrelations associated with the solutions defined by the orthogonal transformation matrices varied from .24 to .48 for the raw solutions and varied from .11 to .48 for the normal solutions. The lowest primary intercorrelations were associated with the orthomax weights of negative three, however the apparrent ideal primary intercorrelation of .48 (Holzinger and Harman,1941) was associated with a number of solutions. As the orthomax weight progressed from negative three toward positive three the associated primaries appeared to become more related until the degree of relationship was approximately .48. Once this particular level of relationship was attained it served as a ceiling inasmuch as all solutions generated from subsequent orthomax weights in the progression toward positive three resulted in a primary intercorrelation of .48.

As with the previous examples the normal varimax and the Harris and Kaiser procedures defined the solutions having the clearest structure. For both sets of solutions it is the oblique independent cluster solution that has the clearest structure.

Example 4 Approximate Orthogonal Independent Clusters(Non-Plasmodal)

The data used for this example define approximate orthogonal independent clusters as might be noted if one were to plot the principal axes representation in Table 3. These data are referred to by Harman (1967) as the five socio-economic variables.

Table 3 about here

Table 3 Selected General Independent Cluster Solution for Illustrative Example 4*.

		Type Solution									
				Harris and Kaiser				Normal Varimax			
		Principal Axis†		Oblique		Orthogonal		Oblique		Orthogonal	
Pattern Matrix	1	66	-74	-18	103	-03	99	-17	103	-02	99
	2	71	62	97	-10	94	04	97	-12	94	02
	3	94	-63	09	111	25	111	11	110	20	110
	4	92	20	76	38	80	49	76	37	81	48
	5	73	64	99	-10	97	04	99	-12	97	03
Primary Intercorrelations				100	28			100	28		
				28	100			28	100		
Associated Orthogonal Transformation Matrix				73	69	73	69	73	68	73	68
				-69	73	-69	73	-68	73	-68	73

* All values have been multiplied by 100 to eliminate decimals.

† Computed from centroid solution.

As with the other illustrative data sets 24 independent cluster solutions were generated along with the Harris and Kaiser solutions. Inasmuch as the normal varimax independent cluster soltuions appeared to result in the best structure they are presented along with the Harris and Kaiser solutions in Table 3. The most obvious conclusion one might draw from observing Table 3 is that the orthogonal independent cluster solutions have more trivial loadings, approximate zero loadings, than do the oblique independent cluster solutions.

As with the previous example the smallest magnitude for the primary intercorrelation was associated with an orthomax weight of negative three. As the orthomax weight approached positive unity the primary intercorrelation stabilized at .28 and all subsequent oblique solutions associated with orthomax weights greater than unity defined primary intercorrelations of .28.

In comparing the Harris and Kaiser solutions with the normal varimax solutions it does not appear as though there is much difference between the two sets of solutions defined. However when one compares the orthonormal transformation matrices used for the two sets of solutions it is readily apparent that there is a difference in the two solution sets. The Harris and Kaiser solutions are influenced slightly more by the first principal axis than are the normal varimax solutions. In the development of the normal varimax Kaiser(1958) noted that the quartimax transformation solutions tended to be influenced by the stronger principal axes. Whether of not this influence is desirable might be debated, however the purpose of this manuscript is not one of debating the desirability of principal axis influences in the orthomax transformation solutions.

Example 5 Non-Independent Cluster Problem(Thurstone Box Problem)

The Thurstone box problem(Thurstone,1947) is a classic plasmodal problem whose solution is known to be something other than an independent cluster solution. The variable complexities range from one to three for this three factor solution. Although no evidence is offered in this manuscript it is likely that if one were to define a span of transformation solutions with the orthogonal independent cluster solution at one extreme and the oblique independent cluster solution at the other extreme the solution to the box problem would be midway between the two independent cluster solutions.

Unlike the other examples the subjective solution as well as three sets of independent cluster solutions are presented for the box problem in Table 4, the Harris and Kaiser solutions, the normal varimax solutions and finally the solutions associated with the transformation matrix determined as a function of maximizing the orthomax criterion for an orthomax weight of negative unity with the reported raw matrix (QMT).

Table 4 about here

For the first two sets of solutions, the Harris and Kaiser and the normal varimax, it is very difficult to determine whether the orthogonal solution has "better" simple structure that its oblique analog. When comparing the Harris and Kaiser solutions with the normal varimax solutions it is evident that they are almost identical solutions.

Table 4 Selected General Independent Cluster Solutions for Illustrative Example 5*.

		Type Solution								
					Harris and Kaiser					
		Principal Axis†			Oblique			Orthogonal		
Pattern Matrix	1	66	64	-40	109	-11	-12	99	05	13
	2	73	06	67	-13	-11	110	13	13	98
	3	66	-68	-28	-12	108	-11	05	98	12
	4	87	35	33	41	-14	80	56	12	81
	5	83	-35	-42	33	91	-17	43	89	14
	6	84	-54	04	-15	84	35	09	87	49
	7	86	51	07	73	-17	51	79	09	61
	8	84	28	-46	90	33	-15	88	44	16
	9	87	-28	42	-19	41	83	09	55	83
	10	88	44	20	59	-16	66	69	11	72
	11	88	-02	-47	67	64	-17	71	69	16
	12	88	-42	25	-19	63	63	08	72	69
	13	67	64	-37	107	-13	-08	98	05	16
	14	72	-01	66	-19	04	108	08	19	96
	15	63	-66	-32	-08	106	-16	06	96	08
	16	94	-30	-07	17	72	29	35	78	48
	17	97	25	-02	61	18	42	72	38	59
	18	62	61	-42	107	-09	-16	96	06	09
	19	71	13	64	-07	-17	106	17	08	95
	20	66	-68	-27	-12	106	-10	05	97	13
Primary Intercorrelations					100	39	50			
					39	100	49			
					50	49	100			
Associated Orthogonal Transformation Matrix					-56	57	60	-56	57	60
					67	-74	08	67	-74	08
					-49	-36	80	-49	-36	80

* All values have been multiplied by 100 to eliminate decimals.

† Computed from centroid solution.

Table 4 Selected General Independent Cluster Solutions for Illustrative Example 5*. (cont.)

		Type Solution					
		Normal Varimax					
		Oblique			Orthogonal		
Pattern Matrix	1	109	-12	-13	99	05	11
	2	-11	-10	109	14	15	97
	3	-12	108	-12	05	98	10
	4	43	-13	78	57	14	81
	5	33	91	-18	43	89	12
	6	-15	86	33	09	87	47
	7	74	-16	49	78	10	60
	8	91	33	-16	89	44	14
	9	-18	42	82	10	57	82
	10	60	-15	64	70	12	71
	11	67	64	-18	71	79	14
	12	-18	65	61	09	73	68
	13	107	-13	-09	98	05	14
	14	-17	03	107	09	20	95
	15	-08	106	-17	06	96	06
	16	17	73	27	36	79	46
	17	62	18	40	72	39	57
	18	107	-10	-17	96	06	07
	19	-05	-15	105	18	10	94
	20.	-12	107	-11	05	98	11
Primary Intercor-relations		100	40	50			
		40	100	49			
		50	49	100			
Associated Orthogonal Transforma-tion Matrix					-57	58	58
					67	-73	08
					-47	-35	81

* All values have been multiplied by 100 to eliminate decimals.

Table 4 Selected General Independent Cluster Solutions for Illustrative Example 5*. (cont.)

Type Solution

Raw Orthomax
Weight = -1.0

		Oblique			Orthogonal			Subjective [1]		
Pattern Matrix	1	113	-15	-25	99	06	-08	100	00	00
	2	10	09	91	31	30	90	03	01	99
	3	-17	112	-22	04	99	-06	01	98	02
	4	61	-01	58	71	26	66	49	00	76
	5	29	93	-31	42	89	-10	40	87	01
	6	-11	96	18	16	94	30	01	82	41
	7	88	-09	31	89	19	42	74	-03	54
	8	92	32	-31	89	46	-09	87	40	02
	9	-04	59	63	23	69	69	-01	46	80
	10	76	-05	45	82	23	55	63	-01	66
	11	66	64	-33	71	70	-11	69	66	01
	12	-08	79	44	20	83	53	-01	65	64
	13	-112	-16	-22	99	06	-05	98	-01	04
	14	03	17	89	26	35	88	-02	06	97
	15	-15	109	-26	05	96	-10	03	96	-02
	16	22	81	10	42	85	26	29	72	38
	17	73	26	21	81	47	36	66	28	48
	18	110	-14	-28	96	06	-11	97	01	-03
	19	16	03	88	35	25	87	07	-04	95
	20	-17	111	-21	04	98	-05	00	98	03
Primary Intercor-relations		100	47	41				100	10	23
		47	100	40				10	100	
		41	40	100				23	22	100
Associated Orthogonal Transforma-tion Matrix		-65	66	37	-65	66	37			
		-69	72	-07	-69	72	-07			
		-31	-21	93	-31	-21	93			

* All values have been multiplied by 100 to eliminate decimals.

1 Thurstone(1947)

Furthermore when the two solution sets are compared to the subjective solution it is apparent that they all define essentially the same factor structure.

The third set of solutions, however, is considerably different from the first two sets. The third solution set defines factors that are not the same as those defined by the first two sets of independent cluster solutions.While the loadings in the first two.sets of solutions define patterns identical to each other and similar to the subjective solution the third set of solutions define patterns that are related but not the same as those defined by the normal varimax solutions and the Harris and Kaiser solutions.

When varying the orthomax weight on raw and row normalized matrices a vast array of different independent cluster solutions were generated. As with the eight physical variables the primaries tended to show a low degree of relationship when associated with orthomax weights less than negative unity. The primary intercorrelations tended toward stability when the orthomax weight was greater than or equal to unity. As with the previous illustrative examples it was the normal varimax independent cluster solutions as well as the Harris and Kaiser independent cluster solutions that defined what appeared to be the best solutions out of all of those that were generated. As one considers the solutions in Table 4 it is important to keep in mind the fact that the ideal simple structure solution for these data is not of an independent cluster nature.

Example 6 Non - Independent Cluster Problem(Coan's Eggs)

The Coan(1959) egg problem is a "semi-plasmodal" problem whose ideal solution must, by virtue of the methods used in defining the variables, be something other than an independent cluster solution. The 21 variables in the problem are a function of six basic spatial measurements taken on 100 chicken eggs falling into one of four egg grading categories; small, medium, large, and jumbo. Ratios were formed between the six spatial measurements to define an additional 15 variables.

Coan provides both an orthogonal and an oblique transformation solution for these data. Interestingly enough there is little comparability between the factors defined by the two solutions. We have included this particular data set in this manuscript because in contrast to the other examples it clearly indicates that the Harris and Kaiser procedure will define solutions that are considerably different from those solutions defined by the normal varimax. Specifically as may be noted in Table 5 the normal varimax solutions are quite different from the Harris and Kaiser solutions for these data.

Table 5 about here

It can be seen in Table 5 that the normal varimax orthogonal solution is strikingly similar to the subjective orthogonal solution but the normal varimax oblique independent cluster solution bears no resemblence to the subjective oblique solution. The normal varimax oblique solution defines essentially the same factors as its orthogonal analog which are not the same factors described by Coan's oblique solution.

It may also be noted in Table 5 that the Harris and Kaiser oblique independent cluster solution defines many of the same factors as does the subjective oblique solution reported by Coan. More importantly is the fact that the Harris and Kaiser orthogonal solution is very

Table 5 Selected General Independent Cluster Solution for Illustrative Example 6*

		Type Solution																	
								Harris and Kaiser											
		Principal Axis†						Oblique						Orthogonal					
Pattern Matrix	1	99	-07	-04	-04	04	-05	74	-05	00	00	27	06	78	-05	08	00	60	07
	2	87	42	11	10	-11	15	-05	24	12	-02	88	23	47	29	02	-02	72	42
	3	88	-35	-19	14	-12	15	00	-02	03	15	96	-34	56	-14	13	21	73	-23
	4	73	59	-30	-03	-10	1C	02	07	-45	-01	78	16	40	21	-46	-01	65	41
	5	96	16	01	03	-24	03	-05	-24	11	03	94	41	55	-01	06	01	78	31
	6	92	-30	-06	-12	-12	19	00	-02	06	-16	100	-37	59	-14	15	-08	75	-19
	7	93	-28	-11	-11	12	-08	97	-09	-07	-01	03	-14	83	-16	08	-01	50	-13
	8	97	08	03	-11	11	-14	102	-07	-01	-06	-06	27	83	00	04	-09	50	22
	9	91	37	09	-03	12	-06	89	01	21	-02	02	-12	80	-12	31	00	47	-14
	10	95	18	-08	-07	22	-06	115	14	-08	-03	-12	-22	87	-02	07	-01	47	-09
	11	97	09	-02	13	09	-13	88	01	03	18	05	29	78	07	05	15	54	23
	12	12	93	33	-04	04	03	02	41	08	-20	-02	59	-01	55	-15	-28	07	77
	13	28	-41	82	25	00	06	-02	32	113	-01	09	12	17	12	97	-01	10	-02
	14	01	84	26	28	26	25	01	116	07	00	-02	-16	-08	84	-12	-01	02	51
	15	13	93	21	27	04	00	-01	46	06	15	00	63	-03	60	-17	06	10	77
	16	01	-97	19	17	-01	00	02	-13	55	16	-03	-42	09	-37	64	21	-05	-64
	17	17	-87	-33	25	13	15	00	20	-08	30	02	106	-02	-26	14	41	-09	-85
	18	01	-28	-38	86	-03	-11	01	00	00	100	00	03	-01	-02	02	97	05	-19
	19	12	80	-55	-05	11	09	00	33	-90	01	04	-15	-13	35	-86	01	01	32
	20	02	95	-28	05	01	-04	-02	15	-56	10	00	45	-10	39	-65	03	05	62
	21	17	90	17	21	-14	22	-02	-16	07	22	-04	122	00	34	-18	08	12	90
Primary Intercorrelations								100	-11	23	03	91	04						
								-11	100	-49	-18	07	87						
								23	-49	100	11	07	-53						
								03	-18	11	100	04	-32						
								91	07	07	04	100	22						
								04	87	-53	-32	22	100						
Associated Orthogonal Transformation Matrix								74	01	09	01	66	12	74	01	09	01	66	12
								-12	48	-45	-13	05	73	-12	48	-45	-13	05	73
								-04	24	85	-34	-13	31	-04	24	85	-34	-13	31
								-11	32	24	90	06	08	-11	32	24	90	06	08
								-54	-55	11	05	54	32	-54	-55	11	05	54	32
								37	-55	00	23	-51	50	37	-55	00	23	-51	50

*All values have been multiplied by 100 to eliminate decimals.

†Computed from centroid solution.

Table 5 Selected General Independent Cluster Solution for Illustrative Example 6 (cont.)

Type Solution

		Normal Varimax Orthogonal						Subjective Oblique						Subjective Orthogonal					
Pattern Matrix	1	99	03	06	-01	-02	-03	76	00	00	04	26	02	99	04	02	-01	03	-06
	2	81	54	03	-01	23	07	-03	39	-01	00	88	02	81	55	06	-03	-22	00
	3	90	-25	08	23	25	03	00	02	01	53	100	00	90	-22	02	25	-23	00
	4	70	52	-45	-01	20	-01	03	-02	61	02	79	02	69	57	-41	-03	-18	-03
	5	92	27	04	01	25	-15	-03	-06	-01	04	97	17	91	28	04	-01	-23	-18
	6	93	-22	10	-07	25	04	00	02	-01	-04	103	-47	93	-21	05	-05	-24	00
	7	97	-21	05	-02	-10	-01	99	-09	03	04	03	-10	97	-19	00	-01	12	-04
	8	97	18	04	-11	-13	-06	105	00	-01	-19	-09	05	96	18	02	-12	15	-10
	9	92	-20	28	-01	-10	02	90	09	-29	04	03	-10	93	-21	23	00	11	-02
	10	97	-09	05	-02	-16	08	116	09	01	04	-18	-12	98	-07	01	-01	18	04
	11	95	23	05	13	-09	-04	116	00	00	04	-23	30	95	20	01	00	19	-10
	12	02	96	-08	-27	00	04	03	50	-01	-62	-06	-02	03	94	01	-32	00	04
	13	21	-02	97	00	02	07	-03	74	128	-04	09	-02	25	-12	95	-01	-02	04
	14	-09	91	-03	03	00	39	-03	110	-03	-02	-06	-15	-07	91	07	-02	00	39
	15	02	99	-10	07	01	05	03	57	01	02	-06	47	02	99	01	01	00	05
	16	06	-78	59	22	-01	02	00	00	-69	47	00	02	07	-82	50	25	01	01
	17	-07	-86	09	43	00	22	-06	00	00	91	03	-07	-07	-84	02	48	-01	22
	18	03	-17	00	98	00	-01	00	02	00	193	-03	152	00	-10	01	99	01	-01
	19	12	52	-82	02	01	13	00	-02	106	04	00	-05	-15	61	-75	00	00	16
	20	07	78	-61	03	01	-02	00	00	72	00	-03	37	-09	84	-52	-01	00	-01
	21	07	95	-13	06	00	-24	03	06	03	02	-06	95	05	96	-04	00	01	-24
Primary Intercorrelations								100	-06	-19	-01	92	03						
								-06	100	65	-72	09	82						
								-19	65	100	-54	-08	61						
								-01	-72	-54	100	-19	-86						
								92	09	-08	-19	100	23						
								03	82	61	-86	23	100						
Associated Orthogonal Transformation Matrix		99	12	08	00	-05	-02												
		-08	91	-39	-13	-02	00												
		-10	32	88	-34	04	00												
		-05	24	27	93	-06	11												
		-05	-01	04	02	75	-66												
		-02	-04	00	-12	-66	74												

*All values have been multiplied by 100 to eliminate decimals.

Table 5 Selected General Independent Cluster Solution for Illustrative Example 6* (cont.)

Type Solution

		Normal Orthomax Weight = 2												Normal Varimax					
		Oblique						Orthogonal						Oblique					
Pattern Matrix	1	76	-05	01	-01	24	06	80	-01	08	00	58	08	101	00	-01	-02	-06	-02
	2	-02	37	09	-03	84	13	49	40	-01	-07	68	34	74	54	11	01	20	04
	3	03	-05	02	20	93	-22	58	-15	13	25	73	-16	87	-28	-02	18	22	04
	4	06	13	-49	-01	73	10	44	31	-48	-06	60	33	70	35	-44	01	16	01
	5	-02	-17	07	01	92	40	57	09	04	-01	76	31	85	25	06	02	21	-14
	6	03	-06	05	-13	98	-30	60	-15	16	-04	75	-15	90	-30	00	-12	24	06
	7	99	-14	-04	-03	01	-10	84	-17	10	01	49	-09	102	-26	-07	-05	-14	01
	8	104	-04	00	-09	-09	20	84	07	05	-11	48	19	100	15	00	-10	-18	-05
	9	90	-01	24	-01	01	-10	80	-12	33	02	46	-10	94	-17	20	-03	-13	03
	10	116	12	-04	-02	-15	-23	88	-02	09	01	45	-08	104	-13	-06	-03	-21	10
	11	91	-08	03	17	01	25	80	14	05	12	51	22	97	26	04	16	-14	-04
	12	02	60	06	-26	-04	32	00	69	-19	-37	04	58	-02	98	13	-20	-02	05
	13	-07	44	117	-01	11	07	14	13	97	00	12	00	07	33	109	01	04	06
	14	01	138	08	01	-07	-41	-07	93	-15	-07	-03	30	-12	100	17	08	-02	39
	15	00	69	03	11	-04	40	-01	75	-22	-04	05	61	-04	108	12	16	-02	04
	16	-01	-22	59	20	00	-24	06	-48	68	29	-01	-48	02	-61	49	16	01	01
	17	-01	06	-04	39	02	-83	-04	-43	18	51	-07	-71	-03	-84	-09	34	02	21
	18	01	05	-03	108	-04	20	00	-03	01	99	03	-07	01	03	-01	101	-03	-06
	19	04	35	-93	02	-01	-24	-10	39	-89	-04	-03	19	-03	28	-82	04	-01	14
	20	01	26	-61	07	-04	30	-07	50	-70	05	00	49	-04	66	-50	09	-02	02
	21	-01	04	02	16	-06	103	02	53	-24	03	07	80	00	104	08	17	-03	25
Primary Intercorrelations		100	-01	20	03	92	10							100	06	16	03	25	-08
		-01	100	-54	-38	10	89							06	100	-49	-27	14	00
		20	-54	100	24	10	-54							16	-49	100	08	-08	02
		03	-38	24	100	03	-45							03	-27	08	100	07	14
		92	10	10	03	100	21							25	14	-08	07	100	-04
		10	89	-54	-45	21	100							-08	00	02	14	-04	100
Associated Orthogonal Transformation Matrix		-76	-07	-09	00	-63	-12	-76	-07	-09	00	-63	-12	99	12	08	00	-05	-02
		-09	62	-50	-22	01	57	-09	62	-50	-22	01	57	-08	91	-39	-13	-02	00
		07	-30	-83	37	11	-26	07	-30	-83	37	11	-26	-10	32	88	-34	04	00
		-10	36	22	89	03	12	-10	36	22	89	03	12	-05	24	27	93	-06	11
		52	44	-06	-02	-57	-45	52	44	-06	-02	-57	-45	-05	-01	04	02	75	-66
		37	-45	01	16	-51	62	37	-45	01	16	-51	62	-02	-04	00	-12	-66	74

*All values have been multiplied by 100 to eliminate decimals.

similar to the subjective orthogonal solution. That is, although the Harris and Kaiser orthogonal and oblique independent cluster solutions bear no resembelance to each other in terms of the factors they define and although they bear little similarity to the normal varimax oblique solution in terms of factors defined they do appear to define the same factors as their corresponding subjective sclutions as reported by Coan.

In Table 5 an additional set of othomax solutions is reported. For these particular data the definition of an ideal solution was somewhat elusive, but if Coan's reported solutions are accepted as the ideal solutions then the normal orthomax solutions associated with an orthomax weight of positive two are more desirable than the normal varimax solutions. These solutions are the extra set reported. It is interesting to note that although these solutions are not the same as the Harris and Kaiser solutions they would result in the same interpretations inasmuch as they define the same factors, orthogonal and oblique, as the corresponding orthogonal and oblique, respectively, Harris and Kaiser independent cluster solutions.

Summary of Empirical Applications

As with so many empirical investigations caution must be exercised in attempting to generalize the system associated with the orthomax independent cluster solutions generated for the illustrative data sets. The systematic varying of the orthomax weight will not always result in a set of independent cluster solutions that appear to be systematically related nor will it result in a stabilization of the primary intercorrelations. Yet, consistancies and inconsistancies noted in the empirical section served as the bases for the title of the manuscript.

Using the systematized computational procedures described in an earlier part of this manuscript a variety of independent cluster solutions were generated. For every data set the equamax, quartimax and varimax solutions, both raw and normal, were computed along with numerous other orthogonal solutions. For each orthogonal solution an oblique analog was computed. Also computed for each data set were the Harris and Kaiser independent cluster solutions.

The solutions recorded were the Harris and Kaiser solutions as well as the "best" set of orthomax solutions. For four out of five of the data sets the normal varimax independent cluster solutions were superior to the other orthomax independent cluster solutions, in simple structure and/or defining the factors in accordance with the subjective solutions. However, it was noted that for one particular set of data the normal varimax solutions tended to be misleading in terms of defining factors. Although there was always a reasonable orthomax solution set for each set of empirical data analyzed the particular orthomax weight defining the solution could not be determined _a priori_. Furthermore it was evident that disasterous results could emerge with certain orthomax solutions.

Fortunately the Harris and Kaiser procedures as modified in this manuscript _always_ provided the best set of independent cluster solutions for the data sets. These results were not expected. The fact that the orthogonal independent cluster solution defined by the Harris and Kaiser procedures was without exception as good or better than the best orthomax orthogonal solution was totally unexpected. It would seem as though we have, quite by accident, discovered a procedure for computing orthogonal transformation solutions that may be superior to the more traditional procedures employed with the general orthomax equation, at least within the framework of simple structure.

For at least one particular data set, the five socio-economic variables, the orthogonal independent cluster solution appeared to be better within a simple structure framework than the oblique independent cluster solution. For another data set, the box problem, it was evident that the ideal solution was not of an independent cluster nature but the orthogonal independent cluster solution did provide a much more reasonable simple structure solution than did the oblique independent cluster solution. For the other data sets the oblique independent cluster solutions appeared to be better within a simple structure framework than the orthogonal independent cluster solutions.

Discussion

The objective of this manuscript was to present a general model for the independent cluster solution in factor analysis. The model, having its basis in the work of Harris and Kaiser (1964), was presented and discussed.

It was noted that for data that define perfect orthogonal independent clusters the model will result in only one solution, an orthogonal independent cluster solution. More precisely the principal axis representation of such a solution would have perfect simple structure. For a data set that define perfect oblique independent clusters there are two possible solutions defined; the first is an orthogonal solution while the second is an oblique analog of the orthogonal solution. For such a data set it is rather apparent, when looking at the transformation solutions, which particular solutions has better simple structure. The simple structure associated with the oblique independent cluster solution will be vastly superior to that of the orthogonal solution.

For a data set that defines factors that are not of an independent cluster nature there are also two possible independent cluster representations; the orthogonal representation, which is the type of transformation solution typically used in many factor analyses, and the oblique analog of the orthogonal solution. For this particular data set the transformation solution that is most interpretable within a simple structure framework will be a function of whether or not the data define factors that tend toward orthogonality or whether or not they define factors that do not tend toward orthogonality. Unfortunately we have been unsuccessful at defining some meaningful criterion for comparing an orthogonal solution with its oblique analog to determine which solution is more interpretable within a simple structure framework.

The results presented in the empirical section suggest that the Harris and Kaiser (1964) procedures as modified in this manuscript will provide the best set of independent cluster solutions. Although it may be possible to generate a number of independent cluster solutions by systematically varying the orthomax weight the results of the empirical applications presented herein suggest that when one does get a desirable solution through variations of the orthomax weight in the orthomax criterion the results that they get will most likely be similar to the results obtained using the modified Harris and Kaiser procedures. Alternatively it is possible that they might get very misleading results.

The orthogonal transformation solution has been a panacea in factor analysis. With the advent of the work of Harris and Kaiser (1964) the oblique independent cluster solution also began to become a panacea for some researchers. Seldom if ever do the researchers report both an orthogonal and an oblique independent cluster solution. Even when the Harris and Kaiser oblique independent cluster procedures appear to define a poor solution it is possible to compute an orthogonal solution that is as bad or worse than the oblique Harris and Kaiser independent cluster solution. Using the procedures set forth in this manuscript it is possible to bring some order to the chaotic use of the independent cluster solution.

In summary then:

a) in this manuscript certain algebraic similarities between the orthogonal and oblique independent cluster solutions have been noted, thereby providing an algebraic link between the orthogonal and oblique independent cluster solutions;

b) empirically it has been demonstrated that by allowing the orthomax weight to vary in the orthomax criterion it is possible to generate a variety of independent cluster solutions for a single data set;

c) although the general model does nost assuredly exist it appears as though it is the Harris and Kaiser procedure that define the best set of independent cluster solutions.

As a function of the empirical presentations in this manuscript the following procedure is suggested for use in the routine calculation of orthogonal and oblique independent cluster solutions from some orthogonal factor matrix $\underline{P}$.

a) Compute the major product $\underline{R}^*$ of $\underline{P}$ as $\underline{PP}'$.

b) Compute the principal axis representation $\underline{QM}$ of $\underline{R}^*$, where the columns of $\underline{Q}$ are unit length latent vectors associated with the non-zero latent roots which are the diagonal entries of $\underline{M}^2$.

c) Compute an orthonormal transformation matrix $\underline{T}$ such that $\underline{QT}$ maximizes the orthomax criterion. The orthomax criterion for this particular case as noted by Harris and Kaiser (1964) will necessarily be general inasmuch as the second term of the criterion equation, the one associated with the orthomax weight, will be a constant.

d) Compute the primary factor intercorrelation matrix $\underline{W}$* as

$$\underline{W}^* = \underline{D^{-1}T'M^2TD^{-1}}$$

where $\underline{D}^2$ is the diagonal matrix whose nonzero entries represent the column sums of squares of $\underline{MT}$.

e) Compute the primary pattern matrix $\underline{A}$* and the primary structure matrix $\underline{B}$*:

$$\underline{A}^* = \underline{QTD} ;$$

$$\underline{B}^* = \underline{QM^2TD^{-1}} .$$

f) Finally compute the orthogonal analog, $\underline{F}$, of the oblique independent cluster solution as:

$$\underline{F} = \underline{QMT} .$$

In final conclusion it is most prudent to realize that any data set may be forced into an independent structure framework. If, when in the independent cluster framework, the variables appear to be complex it may be that the independent cluster framework does not adequately describe the data in a simple structure sense. For such data some other type of oblique solution will most likely provide a better simple structure solution than the independent cluster solutions.

References

Coan, R. W. A comparison of oblique and orthogonal factor solutions. _Journal of Experimental Education,_ 1959, _27_, 151-166.

Harris, C. W. and Kaiser, H. Oblique factor analytic solutions by orthogonal transformation. _Psychometrika,_ 1964, _29_, 347-362.

Harman, H. H. _Modern factor analysis_. Chicago: University of Chicago Press, 1967.

Holzinger, K. J. and Harman, H.H. _Factor analysis: a synthesis of factorial methods_. Chicago: University of Chicago Press, 1941.

Kaiser, H. The varimax criterion for analytic rotation in factor analysis. _Psychometrika_, 1958, _28_, 187-200.

Kaiser, H. Psychometric approaches to factor analysis. In A. Anastasi (Ed.), _Testing problems in perspective_. Washington, American Council on Education, 1966, pp.360-368.

Neuhaus, J. O. and Wrigley, C. The quartimax method; an analytical approach to orthogonal simple structure. _British Journal of Statistical Psychology_, 1954, _7_, 81-91.

Saunders, D. R. Trans-varimax: some properties of the ratiomax and equamaz criteria for blind orthogonal rotation. Paper read at annual American Psychological Association meeting in St. Louis, 1962.

Thurstone, L. L. _Multiple factor Analysis_. (Seventh impression). Chicago University of Chicago Press, 1947.